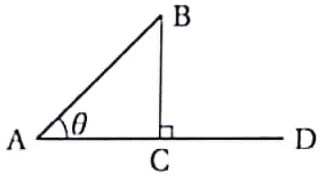
	Exam I	
Name:	Department: Comp. Eng.	GRADE
Student No:	Course: Calculus II	
Signature:	Date: 30/03/2018	

Each problem is worth equal points. Demonstrate your solution steps clearly.

1. Suppose $\overrightarrow{AB} = \sqrt{3}i + 2j - 3k$ and $\overrightarrow{AD} = \sqrt{3}i - 2j - 3k$.
Find the area of triangle ABC.



$|\overrightarrow{AB}| = |\overrightarrow{AD}| = (3+4+9)^{1/2} = 4.$

$\overrightarrow{AB} \cdot \overrightarrow{AD} = |\overrightarrow{AB}||\overrightarrow{AD}| \cos\theta$

$3 - 4 + 9 = 4^2 \cos\theta$

$\cos\theta = \frac{8}{16} = \frac{1}{2}$

$|\sin\theta| = \frac{\sqrt{3}}{2}$

$\text{Area} = \frac{1}{2} |\overrightarrow{BC}||\overrightarrow{AC}|$

$= \frac{1}{2} |\overrightarrow{AB}||\sin\theta| |\overrightarrow{AB}| \cos\theta$

$= \frac{1}{2} \cdot 4 \cdot \frac{\sqrt{3}}{2} \; 4 \cdot \frac{1}{2}$

$= 2\sqrt{3}$

$$\boxed{2\sqrt{3}}$$

2. Write an equation of the plane that contains the point $(3, -4, 2)$ and the vectors $\vec{u} = 2i - j$ and $\vec{v} = 5i + j - k$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 5 & 1 & -1 \end{vmatrix} = i + 2j + 7k$$

$$\boxed{(x-3) + 2(y+4) + 7(z-2) = 0}$$

3. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(2x-3)^n}{n}$.

$$\lim_{n\to\infty} \left| \frac{(2x-3)^{n+1}/(n+1)}{(2x-3)^n/n} \right| = \lim_{n\to\infty} |2x-3| \left(\frac{n}{n+1}\right) = |2x-3|$$

The series converge absolutely for $|2x-3| < 1$, diverges for $|2x-3| > 1$.

$2x - 3 = 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n}$ series diverge. (harmonic series)

$2x - 3 = -1 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ series converge by alternating series test.

$$\boxed{1 \leq x < 2}$$

4. Find the sum of the series $\sum_{k=1}^{\infty} \frac{2^{k+3}}{e^{k-3}}$ if it converges.

$$\sum_{k=1}^{\infty} \frac{2^{k+3}}{e^{k-3}} = \frac{2^3}{e^{-3}} \sum_{k=1}^{\infty} \left(\frac{2}{e}\right)^k = 2^3 e^3 \left(\sum_{k=0}^{\infty} \left(\frac{2}{e}\right)^k - 1\right) = 8e^3 \left(\frac{1}{1-2/e} - 1\right)$$

$$= 8e^3 \left(\frac{e}{e-2} - 1\right) = 8e^3 \left(\frac{e-e+2}{e-2}\right) = \frac{16e^3}{e-2}$$

$$\boxed{\frac{16e^3}{e-2}}$$

5. Find the Taylor series expansion for $f(x) = 5/x^2$ about $a = 1$.

$f(x) = 5/x^2$ $\quad f(1) = 5$

$f'(x) = \frac{5\cdot(-2)}{x^3}$ $\quad f'(1) = 5\cdot(-2)$

$f''(x) = \frac{5\cdot(-2)(-3)}{x^4}$ $\quad f''(1) = 5\cdot(-2)(-3)$

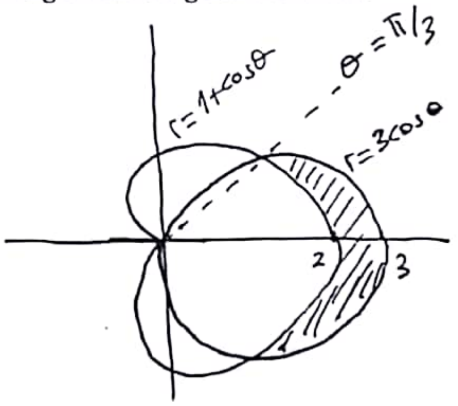
$\vdots$

$f^{(n)}(x) = \frac{5\cdot(-2)(-3)\cdots(-(n+1))}{x^{n+2}}$ $\quad f^{(n)}(1) = 5\cdot(-1)^n(n+1)!$

$a_n = \frac{f^{(n)}(1)}{n!} = 5(-1)^n(n+1)$

$$\boxed{\sum_{n=0}^{\infty} 5(-1)^n(n+1)(x-1)^n}$$

6. Sketch the area of the region that lies inside the curve $r = 3\cos\theta$ and outside the curve $r = 1+\cos\theta$. Write an integral which gives the area.



The curves intersect at:

$3\cos\theta = 1+\cos\theta$

$\cos\theta = 1/2 \Rightarrow \theta = \pi/3$

$\theta = -\pi/3$

$$\boxed{\text{Area} = \frac{1}{2}\int_{-\pi/3}^{\pi/3} \left(9\cos^2\theta - (1+\cos\theta)^2\right) d\theta}$$